CatLog: A Categorial Parser/Theorem-Prover[1]

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Implements all the categorial logic and analyses the author has been concerned with to date.

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demo: cross serial dependencies

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- intercalation $\times$ (displacement) Morrill, Valentín & Fadda (2011)[14]

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$$\mathcal{T} ::= \delta \mid \top \mid \mathcal{T} + \mathcal{T} \mid \mathcal{T}\&\mathcal{T} \mid \mathcal{T} \to \mathcal{T} \mid \mathbf{L}\mathcal{T}$$

Semantic frames

A *semantic frame* comprises a familiy $\{D_\tau\}_{\tau \in \delta}$ of non-empty *basic type domains* and a non-empty set W of worlds. This induces a *type domain* D_τ for each type τ as follows:

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$D_\top$	$=$	$\{\emptyset\}$		singleton set
$D_{\tau_1+\tau_2}$	$=$	$D_{\tau_2} \uplus D_{\tau_1}$	$(\{1\}\times D_{\tau_1})\cup(\{2\}\times D_{\tau_2})$	disjoint union
$D_{\tau_1\&\tau_2}$	$=$	$D_{\tau_1}\times D_{\tau_2}$	$\{\langle m_1, m_2\rangle \mid m_1\in D_{\tau_1}\ \&\ m_2\in D_{\tau_2}\}$	Cartesian product
$D_{\tau_1\to\tau_2}$	$=$	$D_{\tau_2}^{D_{\tau_1}}$	the set of all functions from D_{τ_1} to D_{τ_2}	functional exponentiation
$D_{\mathbf{L}\tau}$	$=$	D_τ^W	the set of all functions from W to D_τ	functional exponentiation

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The sets Φ_τ of *terms* of type τ for each type τ are defined on the basis of sets C_τ of constants of type τ and denumerably infinite sets V_τ of variables of type τ for each type τ as follows:

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Φ_τ	$::=$	C_τ	
Φ_τ	$::=$	V_τ	
$\Phi_\top$	$::=$	d	
Φ_τ	$::=$	$\Phi_{\tau_1+\tau_2} \rightarrow V_{\tau_1}.\Phi_\tau;\ V_{\tau_2}.\Phi_\tau$	case statement
$\Phi_{\tau+\tau'}$	$::=$	$\iota_1\Phi_\tau$	first injection
$\Phi_{\tau'+\tau}$	$::=$	$\iota_2\Phi_\tau$	second injection
Φ_τ	$::=$	$\pi_1\Phi_{\tau\&\tau'}$	first projection
Φ_τ	$::=$	$\pi_2\Phi_{\tau'\&\tau}$	second projection
$\Phi_{\tau\&\tau'}$	$::=$	$(\Phi_\tau, \Phi_{\tau'})$	ordered pair formation
Φ_τ	$::=$	$(\Phi_{\tau'\rightarrow\tau}\ \Phi_{\tau'})$	functional application
$\Phi_{\tau\rightarrow\tau'}$	$::=$	$\lambda V_\tau \Phi_{\tau'}$	functional abstraction
Φ_τ	$::=$	$^\vee\Phi_{\mathbf{L}\tau}$	extensionalization
$\Phi_{\mathbf{L}\tau}$	$::=$	$^\wedge\Phi_\tau$	intensionalization

Valuations

Given a semantic frame, a *valuation* f mapping each constant of type τ into an element of D_τ, an *assignment* g mapping each variable of type τ into an element of D_τ, and a *world* $i \in W$, each term ϕ of type τ receives an interpretation $[\phi]^{g,i} \in D_\tau$ as follows:

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$$
\begin{array}{rcl}
[a]^{g,i} & = & f(a) \text{ for constant } a \in C_\tau \\
[x]^{g,i} & = & g(x) \text{ for variable } x \in V_\tau \\
[d]^{g,i} & = & \emptyset \\
[\phi \rightarrow x.\psi; y.\psi]^{g,i} & = & \left\{ \begin{array}{ll} [\psi]^{(g-\{(x,g(x))\})\cup\{(x,d)\},i} & \text{if } [\phi]^{g,i} = \langle 1, d\rangle \\ [\chi]^{(g-\{(y,g(y))\})\cup\{(y,d)\},i} & \text{if } [\phi]^{g,i} = \langle 2, d\rangle \end{array} \right. \\
[\iota_1\phi]^{g,i} & = & \langle 1, [\phi]^{g,i}\rangle \\
[\iota_2\phi]^{g,i} & = & \langle 2, [\phi]^{g,i}\rangle \\
[\pi_1\phi]^{g,i} & = & \mathbf{fst}([\phi]^{g,i}) \\
[\pi_2\phi]^{g,i} & = & \mathbf{snd}([\phi]^{g,i}) \\
[(\phi, \psi)]^{g,i} & = & \langle [\phi]^{g,i}, [\psi]^{g,i}\rangle \\
[(\phi\ \psi)]^{g,i} & = & [\phi]^{g,i}([\psi]^{g,i}) \\
[\lambda x\phi]^{g,i} & = & d \mapsto [\phi]^{(g-\{(x,g(x))\})\cup\{(x,d)\},i} \\
[{}^{\vee}\phi]^{g,i} & = & [\phi]^{g,i}(i) \\
[{}^{\wedge}\phi]^{g,i} & = & j \mapsto [\phi]^{g,j}
\end{array}
$$

Free substitution

An occurrence of a variable x in a term is called *free* if and only if it does not fall within any part of the term of the form $x.\cdot$ or $\lambda x\cdot$; otherwise it is *bound* (by the closest $x.$ or λx within the scope of which it falls). The result $\phi\{\psi/x\}$ of substituting term ψ (of type τ) for variable x (of type τ) in a term ϕ is the result of replacing by ψ every free occurrence of x in ϕ. We say that ψ is *free for x in ϕ* if and only if no variable in ψ becomes bound in $\phi\{\psi/x\}$. We say that a term is *modally closed* if and only if every occurrence of $\check{}$ occurs within the scope of an $\hat{}$. A modally closed term is denotationally invariant across worlds. We say that a term ψ is *modally free for x in ϕ* if and only if either ψ is modally closed, or no free occurrence of x in ϕ is within the scope of an $\hat{}$.

Semantic conversion laws

$$\phi \rightarrow y.\psi; z.\chi \quad = \quad \phi \rightarrow x.(\psi\{x/y\}); z.\chi$$

if x is not free in ψ and is free for y in ψ

$$\phi \rightarrow y.\psi; z.\chi \quad = \quad \phi \rightarrow y.\psi; x.(\chi\{x/z\})$$

if x is not free in χ and is free for z in χ

$$\lambda y\phi \quad = \quad \lambda x(\phi\{x/y\})$$

if x is not free in ϕ and is free for y in ϕ

α-conversion

$$\iota_1\phi \rightarrow y.\psi; z.\chi \quad = \quad \psi\{\phi/y\}$$

if ϕ is free for y in ψ and modally free for y in ψ

$$\iota_2\phi \rightarrow y.\psi; z.\chi \quad = \quad \chi\{\phi/z\}$$

if ϕ is free for z in χ and modally free for z in χ

$$\pi_1(\phi, \psi) \quad = \quad \phi$$

$$\pi_2(\phi, \psi) \quad = \quad \psi$$

$$(\lambda x\phi\ \psi) \quad = \quad \phi\{\psi/x\}$$

if ψ is free for x in ϕ, and modally free for x in ϕ

$$\check{}\,\hat{}\,\phi \quad = \quad \phi$$

β-conversion

$$(\pi_1\phi, \pi_2\phi) \quad = \quad \phi$$

$$\lambda x(\phi\ x) \quad = \quad \phi$$

if x is not free in ϕ

$$\hat{}\,\check{}\,\phi \quad = \quad \phi$$

if ϕ is modally closed

η-conversion

Categorial Logic

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i.e. which are interpreted as sets of expressions of sort i.

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The homomorphic type map T maps primitive syntactic types into (not necessarily primitive) semantic types.

Multiplicative Basis: Disp. Calculus with Brackets (Db)

Spirit of Lambek (1958)[5]. Brackets: Morrill (1992)[9], Moortgat (1995)[6]. Displacement: Morrill, Valentín & Fadda (2011)[14].

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$$\begin{array}{rcl} \mathcal{F}_i & ::= & [\,]^{-1}\mathcal{F}_i \\ [[\,]^{-1}A] & = & \{s|\ [s] \in [A]\} \\ \mathcal{F}_i & ::= & \langle\,\rangle\mathcal{F}_i \\ [\langle\,\rangle A] & = & \{[s]|\ s \in [A]\} \end{array}$$

$T([\,]^{-1}A) = T(A)$
antibracket
$T(\langle\,\rangle A) = T(A)$
bracket

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[\langle\,\rangle A] & = & \{[s]|\ s \in [A]\} \\
\mathcal{F}_j & ::= & \mathcal{F}_i\backslash\mathcal{F}_{i+j} \\
[A\backslash C] & = & \{s_2|\ \forall s_1 \in [A], s_1{+}s_2 \in [C]\} \\
\mathcal{F}_i & ::= & \mathcal{F}_{i+j}/\mathcal{F}_j \\
[C/B] & = & \{s_1|\ \forall s_2 \in [B], s_1{+}s_2 \in [C]\} \\
\mathcal{F}_{i+j} & ::= & \mathcal{F}_i{\cdot}\mathcal{F}_j \\
[A{\cdot}B] & = & \{s_1{+}s_2|\ s_1 \in [A]\ \&\ s_2 \in [B]\} \\
\mathcal{F}_0 & ::= & I \\
[I] & = & \{0\}
\end{array}
$$

$T([\,]^{-1}A) = T(A)$
antibracket

$T(\langle\,\rangle A) = T(A)$
bracket

$T(A\backslash C) = T(A) \to T(C)$
under

$T(C/B) = T(B) \to T(C)$
over

$T(A{\cdot}B) = T(A)\&T(B)$
product

$T(I) = \top$
product unit

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$\mathcal{F}_i$	$::=$	$[\,]^{-1}\mathcal{F}_i$	$T([\,]^{-1}A) = T(A)$
$[[\,]^{-1}A]$	$=$	$\{s \mid [s] \in [A]\}$	antibracket
$\mathcal{F}_i$	$::=$	$\langle\,\rangle\mathcal{F}_i$	$T(\langle\,\rangle A) = T(A)$
$[\langle\,\rangle A]$	$=$	$\{[s] \mid s \in [A]\}$	bracket
$\mathcal{F}_j$	$::=$	$\mathcal{F}_i \backslash \mathcal{F}_{i+j}$	$T(A\backslash C) = T(A) \to T(C)$
$[A\backslash C]$	$=$	$\{s_2 \mid \forall s_1 \in [A], s_1{+}s_2 \in [C]\}$	under
$\mathcal{F}_i$	$::=$	$\mathcal{F}_{i+j}/\mathcal{F}_j$	$T(C/B) = T(B) \to T(C)$
$[C/B]$	$=$	$\{s_1 \mid \forall s_2 \in [B], s_1{+}s_2 \in [C]\}$	over
$\mathcal{F}_{i+j}$	$::=$	$\mathcal{F}_i{\cdot}\mathcal{F}_j$	$T(A{\cdot}B) = T(A)\&T(B)$
$[A{\cdot}B]$	$=$	$\{s_1{+}s_2 \mid s_1 \in [A]\ \&\ s_2 \in [B]\}$	product
$\mathcal{F}_0$	$::=$	I	$T(I) = \top$
$[I]$	$=$	$\{0\}$	product unit
$\mathcal{F}_j$	$::=$	$\mathcal{F}_{i+1}{\downarrow_k}\mathcal{F}_{i+j}$	$T(A{\downarrow_k}C) = T(A) \to T(B)$
$[A{\downarrow_k}C]$	$=$	$\{s_2 \mid \forall s_1 \in [A], s_1 \times_k s_2 \in [C]\}$	infix
$\mathcal{F}_{i+1}$	$::=$	$\mathcal{F}_{i+j}{\uparrow_k}\mathcal{F}_j$	$T(C{\uparrow_k}B) = T(B) \to T(C)$
$[C{\uparrow_k}B]$	$=$	$\{s_1 \mid \forall s_2 \in [B], s_1 \times_k s_2 \in [C]\}$	extract
$\mathcal{F}_{i+j}$	$::=$	$\mathcal{F}_{i+1}\odot_k\mathcal{F}_j$	$T(A{\odot}B) = T(A)\&T(B)$
$[A\odot_k B]$	$=$	$\{s_1 \times_k s_2 \mid s_1 \in [A]\ \&\ s_2 \in [B]\}$	wrap
$\mathcal{F}_1$	$::=$	J	$T(J) = \top$
$[J]$	$=$	$\{1\}$	wrap unit

Configurations

The set $\mathcal{O}$ of *configurations* is defined as follows, where Λ is the empty string and $[]$ is the metalinguistic placeholder:

$$\mathcal{O} ::= \Lambda \mid [] \mid \mathcal{F}_0 \mid \mathcal{F}_{i+1}\{\underbrace{\mathcal{O} : \ldots : \mathcal{O}}_{i+1\ \mathcal{O}\text{'s}}\} \mid \mathcal{O}, \mathcal{O} \mid [\mathcal{O}]$$

The sort of a configuration Γ is the number of (metalinguistic) placeholders it contains. Where Δ is a configuration of sort at least 1 and Γ is a configuration, $\Delta|_{>}\Gamma$ ($\Delta|_{<}\Gamma$) is the configuration resulting from replacing the leftmost (rightmost) placeholder in Δ by Γ. The *figure* $\overrightarrow{A}$ of a type A is defined by:

$$(1)\quad \overrightarrow{A} = \begin{cases} A & \text{if } sA = 0 \\ A\{\underbrace{[] : \ldots : []}_{sA\ []\text{'s}}\} & \text{if } sA > 0 \end{cases}$$

Configurations (cont.)

The usual configuration distinguished occurrence notation $\Delta(\Gamma)$ signifies a configuration Δ with a distinguished subconfiguration Γ, i.e. a configuration occurrence Γ with (external) context Δ. In the hypersequent calculus the distinguished *hyperoccurrence* notation $\Delta\langle\Gamma\rangle$ signifies a configuration *hyper*occurrence Γ with external *and internal* context Δ as follows: where Γ is a configuration of sort i and $\Delta_1, \ldots, \Delta_i$ are configurations, the *fold* $\Gamma \otimes \langle\Delta_1, \ldots, \Delta_i\rangle$ is the result of replacing the successive placeholders in Γ by $\Delta_1, \ldots, \Delta_i$ respectively; the *distinguished hyperoccurrence* notation $\Delta\langle\Gamma\rangle$ represents $\Delta_0(\Gamma \otimes \langle\Delta_1, \ldots, \Delta_i\rangle)$.

A *sequent* $\Gamma \Rightarrow A$ comprises an antecedent configuration Γ of sort i and a succedent type A of sort i. The types which are allowed to enter into the antecedent are the input ($^\bullet$) types and the types which are allowed to enter into the succedent are the output (^{o}) types.

Db Calculus, pt. 1/3

$$\frac{}{\overrightarrow{A}\colon x \Rightarrow A\colon x}\ id$$

$$\frac{\Delta\langle \overrightarrow{A}\colon x\rangle \Rightarrow B\colon \psi}{\Delta\langle \overrightarrow{[[\,]^{-1}A}\colon x]\rangle \Rightarrow B\colon \psi}\ [\,]^{-1}L \qquad \frac{[\Gamma] \Rightarrow A\colon \phi}{\Gamma \Rightarrow [\,]^{-1}A\colon \phi}\ [\,]^{-1}R$$

$$\frac{\Delta\langle [\overrightarrow{A}\colon x]\rangle \Rightarrow B\colon \psi}{\Delta\langle \overrightarrow{\langle\,\rangle A}\colon x\rangle \Rightarrow B\colon \psi}\ \langle\,\rangle L \qquad \frac{\Gamma \Rightarrow A\colon \phi}{[\Gamma] \Rightarrow \langle\,\rangle A\colon \phi}\ \langle\,\rangle R$$

Db Calculus, pt. 2/3

$$\frac{\Gamma \Rightarrow A\colon \phi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \Gamma, \overrightarrow{A\backslash C}\colon y\rangle \Rightarrow D\colon \omega[(y\ \phi)/z]}\backslash L \qquad \frac{\overrightarrow{A}\colon x, \Gamma \Rightarrow C\colon \chi}{\Gamma \Rightarrow A\backslash C\colon \lambda x\chi}\backslash R$$

$$\frac{\Gamma \Rightarrow B\colon \psi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{C/B}\colon x, \Gamma\rangle \Rightarrow D\colon \omega[(x\ \psi)/z]}/L \qquad \frac{\Gamma, \overrightarrow{B}\colon y \Rightarrow C\colon \chi}{\Gamma \Rightarrow C/B\colon \lambda y\chi}/R$$

$$\frac{\Delta\langle \overrightarrow{A}\colon x, \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{A{\cdot}B}\colon z\rangle \Rightarrow D\colon \omega[\pi_1 z/x, \pi_2 z/y]}\cdot L \qquad \frac{\Gamma_1 \Rightarrow A\colon \phi \qquad \Gamma_2 \Rightarrow B\colon \psi}{\Gamma_1, \Gamma_2 \Rightarrow A{\cdot}B\colon (\phi, \psi)}\cdot R$$

$$\frac{\Delta\langle \Lambda\rangle \Rightarrow A\colon \phi}{\Delta\langle \overrightarrow{I}\colon t\rangle \Rightarrow A\colon \phi} IL \qquad \frac{}{\Lambda \Rightarrow I\colon d} IR$$

Db Calculus, pt. 3/3

$$\frac{\Gamma \Rightarrow A\colon \phi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{\Gamma|_k A{\downarrow_k}C}\colon y\rangle \Rightarrow D\colon \omega[(y\ \phi)/z]}\downarrow_k L \qquad \frac{\overrightarrow{A}\colon x|_k\Gamma \Rightarrow C\colon \chi}{\Gamma \Rightarrow A{\downarrow_k}C\colon \lambda x\chi}\downarrow_k R$$

$$\frac{\Gamma \Rightarrow B\colon \psi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{C{\uparrow_k}B}\colon x|_k\Gamma\rangle \Rightarrow D\colon \omega[(x\ \psi)/z]}\uparrow_k L \qquad \frac{\Gamma|_k\overrightarrow{B}\colon y \Rightarrow C\colon \chi}{\Gamma \Rightarrow C{\uparrow_k}B\colon \lambda y\chi}\uparrow_k R$$

$$\frac{\Delta\langle \overrightarrow{A}\colon x|_k\overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{A\odot_k B}\colon z\rangle \Rightarrow D\colon \omega[\pi_1 z/x, \pi_2 z/y]}\odot_k L \qquad \frac{\Gamma_1 \Rightarrow A\colon \phi \qquad \Gamma_2 \Rightarrow B\colon \psi}{\Gamma_1|_k\Gamma_2 \Rightarrow A\odot_k B\colon (\phi, \psi)}\odot_k R$$

$$\frac{\Delta\langle[\,]\rangle \Rightarrow A\colon \phi}{\Delta\langle \overrightarrow{J}\colon t\rangle \Rightarrow A\colon \phi}JL \qquad \frac{}{[\,] \Rightarrow J\colon d}JR$$

Additives

Lambek (1961)[4], Girard (1987)[2], van Benthem (1991)[16], Morrill (1990; 1994, ch. 6)[7][15], Kanazawa (1992)[3].

$$
\begin{array}{rcll}
\mathcal{F}_i^p & ::= & \mathcal{F}_i^p \& \mathcal{F}_i^p & T(A\&B) = T(A)\&T(B) \\
[A\&B] & \approx & [A] \cap [B] & \text{additive conjunction} \\
\mathcal{F}_i^p & ::= & \mathcal{F}_i^p + \mathcal{F}_i^p & T(A+B) = T(A) + T(B) \\
[A+B] & \approx & [A] \cup [B] & \text{additive disjunction}
\end{array}
$$

Additives (cont.)

$$\frac{\Gamma\langle\overrightarrow{A}\colon x\rangle \Rightarrow D\colon \omega}{\Gamma\langle\overrightarrow{A\&B}\colon z\rangle \Rightarrow D\colon \omega[\pi_1 z/x]}\&L_1 \qquad \frac{\Gamma\langle\overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\Gamma\langle\overrightarrow{A\&B}\colon z\rangle \Rightarrow D\colon \omega[\pi_2 z/y]}\&L_2$$

$$\frac{\Gamma \Rightarrow A\colon \phi \qquad \Gamma \Rightarrow B\colon \psi}{\Gamma \Rightarrow A\&B\colon (\phi, \psi)}\&R$$

$$\frac{\Gamma\langle\overrightarrow{A}\colon x\rangle \Rightarrow D\colon \phi \qquad \Gamma\langle\overrightarrow{B}\colon y\rangle \Rightarrow D\colon \psi}{\Gamma\langle\overrightarrow{A{+}B}\colon z\rangle \Rightarrow D\colon z \rightarrow x.\phi; y.\psi}+L$$

$$\frac{\Gamma \Rightarrow A\colon \phi}{\Gamma \Rightarrow A{+}B\colon \iota_1\phi}+R_1 \qquad \frac{\Gamma \Rightarrow B\colon \psi}{\Gamma \Rightarrow A{+}B\colon \iota_2\psi}+R_2$$

Semantically inert additives

Morrill (1994, ch. 6)[15]

$$\begin{array}{rcll} \mathcal{F}_i^p & ::= & \mathcal{F}_i^p \sqcap \mathcal{F}_i^p & T(A \sqcap B) = T(A) = T(B) \\ [A \sqcap B] & \approx & [A] \cap [B] & \text{semantically inert additive conjunction} \\ \mathcal{F}_i^p & ::= & \mathcal{F}_i^p \sqcup \mathcal{F}_i^p & T(A \sqcup B) = T(A) = T(B) \\ [A \sqcup B] & \approx & [A] \cup [B] & \text{semantically inert additive disjunction} \end{array}$$

Semantically inert additives (cont.)

$$\frac{\Gamma\langle\overrightarrow{A}\colon z\rangle \Rightarrow D\colon \omega}{\Gamma\langle\overrightarrow{A \sqcap B}\colon z\rangle \Rightarrow D\colon \omega}\sqcap L_1 \qquad \frac{\Gamma\langle\overrightarrow{B}\colon z\rangle \Rightarrow D\colon \omega}{\Gamma\langle\overrightarrow{A \sqcap B}\colon z\rangle \Rightarrow D\colon \omega}\sqcap L_2$$

$$\frac{\Gamma \Rightarrow A\colon \chi \qquad \Gamma \Rightarrow B\colon \chi}{\Gamma \Rightarrow A \sqcap B\colon \chi}\sqcap R$$

$$\frac{\Gamma\langle\overrightarrow{A}\colon z\rangle \Rightarrow D\colon \chi \qquad \Gamma\langle\overrightarrow{B}\colon z\rangle \Rightarrow D\colon \chi}{\Gamma\langle\overrightarrow{A \sqcup B}\colon z\rangle \Rightarrow D\colon \chi}\sqcup L$$

$$\frac{\Gamma \Rightarrow A\colon \chi}{\Gamma \Rightarrow A \sqcup B\colon \chi}\sqcup R_1 \qquad \frac{\Gamma \Rightarrow B\colon \chi}{\Gamma \Rightarrow A \sqcup B\colon \chi}\sqcup R_2$$

Difference

Cf. Morrill & Valentín (2010)[12]

$$\mathcal{F}_i{}^o \quad ::= \quad \mathcal{F}_i{}^o - \mathcal{F}_i{}^o \qquad T(A - B) \quad = \quad T(A)$$

$$\frac{\Gamma \Rightarrow A\colon \phi \qquad \not\vdash \Gamma \Rightarrow B\colon \_}{\Gamma \Rightarrow A - B\colon \phi} -R$$

Modalities

Morrill (1990)[8]

$$
\begin{array}{llllll}
\mathcal{F}_i^p & ::= & \Box\mathcal{F}_i^p & T(\Box A) & = & \mathbf{L}A \quad \text{modality} \\
\mathcal{F}_i^p & ::= & \blacksquare\mathcal{F}_i^p & T(\blacksquare A) & = & \mathbf{L}A \quad \text{modality (rigid designator)}
\end{array}
$$

$$
\frac{\Gamma\langle\overrightarrow{A}: x\rangle \Rightarrow C: \chi}{\Gamma\langle\overrightarrow{\Box A}: y\rangle \Rightarrow C: \chi[{}^{\vee}y/x]}\Box L \qquad \frac{\Box\Gamma \Rightarrow A: \phi}{\Box\Gamma \Rightarrow \Box A: {}^{\wedge}\phi}\Box R
$$

$$
\frac{\Gamma\langle\overrightarrow{A}: x\rangle \Rightarrow C: \chi}{\Gamma\langle\overrightarrow{\blacksquare A}: x\rangle \Rightarrow C: \chi}\blacksquare L \qquad \frac{\blacksquare\Gamma \Rightarrow A: \phi}{\blacksquare\Gamma \Rightarrow \blacksquare A: \phi}\blacksquare R
$$

Exponentials, pt. 1/2

$\mathcal{F}_0{}^\bullet$	$::=$	$!\mathcal{A}_0$	$T(!A)$	$=$	$T(A)$	structural modality Girard (1987)[2], Barry et al. (1991
$\mathcal{F}_0{}^o$	$::=$	$\mathcal{F}_0{}^{o+}$	$T(A^+)$	$=$	$\mathit{list}(T(A))$	Kleene plus Morrill (1994)[15]

Exponentials, pt. 2/2

$$\frac{\Gamma(A\colon x) \Rightarrow B\colon \psi}{\Gamma(!A\colon x) \Rightarrow B\colon \psi}\ !L$$

$$\frac{\Delta(!A\colon x, \Gamma) \Rightarrow B\colon \psi}{\Delta(\Gamma, !A\colon x) \Rightarrow B\colon \psi}\ !P \qquad \frac{\Delta(\Gamma, !A\colon x) \Rightarrow B\colon \psi}{\Delta(!A\colon x, \Gamma) \Rightarrow B\colon \psi}\ !P$$

$$\frac{\Delta(!A\colon x, [!A\colon x, \Gamma]) \Rightarrow B\colon \psi}{\Delta(!A\colon x, \Gamma) \Rightarrow B\colon \psi}\ !C \qquad \frac{\Delta([\Gamma, !A\colon x], !A\colon x) \Rightarrow B\colon \psi}{\Delta(\Gamma, !A\colon x) \Rightarrow B\colon \psi}\ !C$$

$$\frac{\Gamma \Rightarrow A\colon \phi}{\Gamma \Rightarrow A^{+}\colon (\phi, d)}\ {}^{+}R \qquad \frac{\Gamma \Rightarrow A\colon \phi \qquad \Delta \Rightarrow A^{+}\colon \psi}{\Gamma, \Delta \Rightarrow A^{+}\colon (\phi, \psi)}\ {}^{+}R$$

(Semantically Inert) First-order quantification

$\mathcal{F}_i^p$	$::=$	$\forall X \mathcal{F}_i^p$	$T(\forall x A)$	$=$	$T(A)$	1st order univ. qu. Morrill (1994)[15]
$\mathcal{F}_i^p$	$::=$	$\exists X \mathcal{F}_i$	$T(\exists x A)$	$=$	$T(A)$	1st order exist. qu. Morrill (1994)[15]

$$\frac{\Gamma\langle \overrightarrow{A[t/x]}\colon z\rangle \Rightarrow B\colon \psi}{\Gamma\langle \overrightarrow{\forall x A}\colon z\rangle \Rightarrow B\colon \psi}\,\forall L \qquad \frac{\Gamma \Rightarrow A[a/x]\colon \phi}{\Gamma \Rightarrow \forall x A\colon \phi}\,\forall R^*$$

$$\frac{\Gamma\langle \overrightarrow{A[a/x]}\colon z\rangle \Rightarrow B\colon \phi}{\Gamma\langle \overrightarrow{\exists x A}\colon z\rangle \Rightarrow B\colon \phi}\,\exists L^* \qquad \frac{\Gamma \Rightarrow A[t/x]\colon \phi}{\Gamma \Rightarrow \exists x A\colon \phi}\,\exists R$$

* no *a* in conclusion

Synthetic Connectives

In addition to the primitive connectives we may define synthetic connectives which do not extend expressivity, but which permit abbreviations. Unary synthetic connectives are:[2]

$\triangleright^{-1}A$	$=_{df}$	$J\backslash A$	$\{s \mid 1{+}s \in A\}$	$T(\triangleright^{-1}A)$	$=$	$T(A)$	right projection Morrill, Valentín & Fadda (2009)[13]
$\triangleleft^{-1}A$	$=_{df}$	A/J	$\{s \mid s{+}1 \in A\}$	$T(\triangleleft^{-1}A)$	$=$	$T(A)$	left projection Morrill, Valentín & Fadda (2009)[13]
$\triangleright A$	$=_{df}$	$J{\cdot}A$	$\{1{+}s \mid s \in A\}$	$T(\triangleright A)$	$=$	$T(A)$	right injection Morrill, Valentín & Fadda (2009)[13]
$\triangleleft A$	$=_{df}$	$A{\cdot}J$	$\{s{+}1 \mid s \in A\}$	$T(\triangleleft A)$	$=$	$T(A)$	left injection Morrill, Valentín & Fadda (2009)[13]
$\check{}^{k} A$	$=_{df}$	$A{\uparrow}_k I$	$\{s \mid s \times_k 0 \in A\}$	$T(\check{}^{k} A)$	$=$	$T(A)$	split Morrill & Merenciano (1996)[10]
$\hat{}^{k} A$	$=_{df}$	$A{\odot}_k I$	$\{s \times_k 0 \mid s \in A\}$	$T(\hat{}^{k} A)$	$=$	$T(A)$	bridge Morrill & Merenciano (1996)[10]

[2] By unicity of the insertion point, when $sA = 0$, $\check{}^{>}A = \check{}^{<}A$ and we write just $\check{}A$, and when $sA = 1$, $\hat{}^{>}A = \hat{}^{<}A$ and we write just $\hat{}A$.

Synthetic connectives (cont.)

Continuous and discontinuous nondeterministic connectives (Morrill & Valentín 2010[11]) which are binary synthetic connectives are thus, where $+(s_1, s_2, s_3)$ if and only if $s_3 = s_1{+}s_2$ or $s_3 = s_2{+}s_1$, and $\times(s_1, s_2, s_3)$ if and only if $s_3 = s_1 \times_> s_2$ or $s_3 = s_2 \times_< s_1$:

$\frac{B}{A}$	$(A\backslash B) \sqcap (B/A)$	$\{s \mid \forall s' \in A, s_3, +(s, s', s_3) \Rightarrow s_3 \in B\}$	$T(\frac{B}{A})$	$=$	$T(A) \to T(B)$	nondet. d
$A \otimes B$	$(A{\cdot}B) \sqcup (B{\cdot}A)$	$\{s_3 \mid \exists s_1 \in A, s_2 \in B, +(s_1, s_2, s_3)\}$	$T(A \otimes B)$	$=$	$T(A)\&T(B)$	nondet. p
$A{\Downarrow}C$	$(A{\downarrow_>}C) \sqcap (A{\downarrow_>}C)$	$\{s_2 \mid \forall s_1 \in A, s_3, \times(s_1, s_2, s_3) \Rightarrow s_3 \in C\}$	$T(A{\Downarrow}C)$	$=$	$T(A) \to T(C)$	nondet. in
$C{\Uparrow}B$	$(C{\uparrow_>}B) \sqcap (C{\uparrow_<}B)$	$\{s_1 \mid \forall s_2 \in B, s_3, \times(s_1, s_2, s_3) \Rightarrow s_3 \in C\}$	$T(C{\Uparrow}B)$	$=$	$T(B) \to T(C)$	nondet. e
$A{\circledcirc}B$	$(A{\odot_>}B) \sqcup (A{\odot_<}B)$	$\{s_3 \mid \exists s_1 \in A, s_2 \in B, \times(s_1, s_2, s_3)\}$	$T(A{\circledcirc}B)$	$=$	$T(A)\&T(B)$	nondet. d

Synthetic Rules, pt. 1/4

$$\frac{\Gamma\langle \overrightarrow{A}: x\rangle \Rightarrow B: \psi}{\Gamma\langle \overrightarrow{\triangleleft^{-1}A}: x, [\,]\rangle \Rightarrow B: \psi} \triangleleft^{-1}L \qquad \frac{\Gamma, [\,] \Rightarrow A: \phi}{\Gamma \Rightarrow \triangleleft^{-1}A: \phi} \triangleleft^{-1}R$$

$$\frac{\Gamma\langle \overrightarrow{A}: x, [\,]\rangle \Rightarrow B: \psi}{\Gamma\langle \overrightarrow{\triangleleft A}: x\rangle \Rightarrow B: \psi} \triangleleft L \qquad \frac{\Gamma \Rightarrow A: \phi}{\Gamma, [\,] \Rightarrow \triangleleft A: \phi} \triangleleft R$$

$$\frac{\Gamma\langle \overrightarrow{A}: x\rangle \Rightarrow B: \psi}{\Gamma\langle [\,], \overrightarrow{\triangleright^{-1}A}: x\rangle \Rightarrow B: \psi} \triangleright^{-1}L \qquad \frac{[\,], \Gamma \Rightarrow A: \phi}{\Gamma \Rightarrow \triangleright^{-1}A: \phi} \triangleright^{-1}R$$

$$\frac{\Gamma\langle [\,], \overrightarrow{A}: x\rangle \Rightarrow B: \psi}{\Gamma\langle \overrightarrow{\triangleright A}: x\rangle \Rightarrow B: \psi} \triangleright L \qquad \frac{\Gamma \Rightarrow A: \phi}{[\,], \Gamma \Rightarrow \triangleright A: \phi} \triangleright R$$

Synthetic Rules, pt. 2/4

$$\frac{\Delta\langle\overrightarrow{B}\colon y\rangle \Rightarrow C\colon \chi}{\Delta\langle\overrightarrow{\check{}^{k}B}\colon y|_k\Lambda\rangle \Rightarrow C\colon \chi}\ \check{}^{k}L \qquad \frac{\Delta|_k\Lambda \Rightarrow B\colon \psi}{\Delta \Rightarrow \check{}^{k}B\colon \psi}\ \check{}^{k}R$$

$$\frac{\Delta\langle\overrightarrow{B}\colon y|_k\Lambda\rangle \Rightarrow C\colon \chi}{\Delta\langle\overrightarrow{\hat{}^{k}B}\colon y\rangle \Rightarrow C\colon \chi}\ \hat{}^{k}L \qquad \frac{\Delta \Rightarrow B\colon \psi}{\Delta|_k\Lambda \Rightarrow \hat{}^{k}B\colon \psi}\ \hat{}^{k}R$$

Synthetic Rules, pt. 3/4

$$\frac{\Gamma \Rightarrow A\colon \phi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \Gamma, \frac{\overrightarrow{C}}{A}\colon y\rangle \Rightarrow D\colon \omega[(y\ \phi)/z]} -L_1 \qquad \frac{\Gamma \Rightarrow A\colon \psi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \frac{\overrightarrow{C}}{A}\colon x, \Gamma\rangle \Rightarrow D\colon \omega[(x\ \psi)/z]} -L_2$$

$$\frac{\overrightarrow{A}\colon x, \Gamma \Rightarrow C\colon \chi \qquad \Gamma, \overrightarrow{A}\colon x \Rightarrow C\colon \chi}{\Gamma \Rightarrow \frac{C}{A}\colon \lambda x \chi} -R$$

$$\frac{\Delta\langle \overrightarrow{A}\colon x, \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega \qquad \Delta\langle \overrightarrow{B}\colon y, \overrightarrow{A}\colon x\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{A \otimes B}\colon z\rangle \Rightarrow D\colon \omega[\pi_1 z/x, \pi_2 z/y]} \otimes L$$

$$\frac{\Gamma_1 \Rightarrow A\colon \phi \qquad \Gamma_2 \Rightarrow B\colon \psi}{\Gamma_1, \Gamma_2 \Rightarrow A \otimes B\colon (\phi, \psi)} \otimes R_1 \qquad \frac{\Gamma_1 \Rightarrow B\colon \psi \qquad \Gamma_2 \Rightarrow A\colon \phi}{\Gamma_1, \Gamma_2 \Rightarrow A \otimes B\colon (\phi, \psi)} \otimes R_2$$

Synthetic Rules, pt. 4/4

$$\frac{\Gamma \Rightarrow A\colon \phi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \Gamma|_k \overrightarrow{A{\Downarrow}C}\colon y\rangle \Rightarrow D\colon \omega[(y\ \phi)/z]}\Downarrow L \qquad \frac{\overrightarrow{A}\colon x|_>\Gamma \Rightarrow C\colon \chi \qquad \overrightarrow{A}\colon x|_<\Gamma \Rightarrow C\colon \chi}{\Gamma \Rightarrow A{\Downarrow}C\colon \lambda x\chi}\Downarrow R$$

$$\frac{\Gamma \Rightarrow B\colon \psi \qquad \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{C{\Uparrow}B}\colon x|_k\Gamma\rangle \Rightarrow D\colon \omega[(x\ \psi)/z]}\Uparrow L \qquad \frac{\Gamma|_> \overrightarrow{B}\colon y \Rightarrow C\colon \chi \qquad \Gamma|_< \overrightarrow{B}\colon y \Rightarrow C\colon \chi}{\Gamma \Rightarrow C{\Uparrow}B\colon \lambda y\chi}\Uparrow R$$

$$\frac{\Delta\langle \overrightarrow{A}\colon x|_> \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega \qquad \Delta\langle \overrightarrow{A}\colon x|_< \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\Delta\langle \overrightarrow{A \circledcirc B}\colon z\rangle \Rightarrow D\colon \omega[\pi_1 z/x, \pi_2 z/y]}\circledcirc L \qquad \frac{\Gamma_1 \Rightarrow A\colon \phi \qquad \Gamma_2 \Rightarrow B\colon \psi}{\Gamma_1|_k\Gamma_2 \Rightarrow A\circledcirc B\colon (\phi, \psi)}\circledcirc$$

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