The Game of Assertion

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Objections to the “proofs as meanings” account

A. Ranta, 1994:

- *There is no fully presenting expression for the continent of Africa, say. Even the longest encyclopedic text will leave an infinity of properties open, and it is a puzzling question what expression, if any, would determine Africa as a continent.*
- *To define a set is to give exhaustive introduction rules that generate its canonical elements. But* man*, or* tree*, does not admit of such rules. Hence* man *and* tree *are not sets, and cannot serve as domains of quantification in predicate calculus.*

What is a proof of an elementary proposition?

We never know:

- whether all the presuppositions on which a given statement is dependent are fulfilled (there may be an infinity of such presuppositions, or in any way, the list is open)
- according to which rules we are allowed to introduce elements of collections of which we are not sure they are really sets

Objections to a “referentialist” account

- *I am working on...*
- *Your book?*
- *No, my article.*

It is not necessary that the alleged book of the first speaker exists. It may have no denotation at all, in any possible world.

From a referentialist account to an inferentialist one

1. "Proofs" $\longrightarrow$ something more flexible ("paraproofs")
2. Language is mainly dialectical processes $\rightarrow$ the need for an interaction framework

Language, as shown by **R. Brandom**, is mainly *dialectic processes*, and even *social* ones, where "social" here is meant in a very broad sense, in the sense that we may say that *objectivity of thought comes from dialogue and interaction between locutors*.

Brandom's Theory of Inferentialism

- according to Brandom (*Articulating reasons*), "saying things" amounts to entering some special activity, the main component of which consists of being able to **draw inferences**, undertaking **responsibility to entitle oneself to some commitments**, responding to **possible objections** coming from other speakers.
- Sellars: for a response, to have conceptual content is just for it to play a role in the inferential game of *making claims* and *giving and asking for reasons*

Scorekeeping

- "understanding a speech act - grasping its discursive significance - is being able to attribute the right commitments in response. This is knowing **how it changes the *score* of what the performer and the audience are committed and entitled to".**

This amounts to assuming that every assertion comes like a *move* in a play and participants (either real or virtual) are constantly *keeping the score* of the current game at hand.

Asserting

Telling:

The swatch is red

is not simply submitting a proposition to an evaluation by “true” or “false”, but

- *playing it as a token in a game*,
- knowing that other players can ask for reasons for saying it,
  - either by challenging the choice of the name “swatch”
  - or by contesting that “it is red”.
- It is only after the game has come to an end that the assertion can be evaluated.

Propositional content and Representational content

- "We typically think about **inference** solely in terms of the relation between premise and conclusion, that is, as a **monological** relation among propositional contens".
- "**Discursive practice**, *the giving and asking for reasons*, however, involves both inter*content* and inter*personal* relations".
- "The claim is that the representational aspect of the propositional contents that play the inferential roles of premise and conclusion *should be understood* in terms of the **social or dialogical** dimension of communicating reasons, of assessing the significance of reasons offered by others"

Game and score

Suppose we have a set of counters or markers such that producing or playing one has the social significance of making an assertional move in the game. We can call such counters 'sentences'. Then for any player at any time there must be a way of partitionning sentences into two classes, by distinguishing somehow those that he is disposed or otherwise prepared to assert (perhaps when suitably prompted). These counters, which are distinguished by bearing the player's mark, being on his list, or being kept in his box, constitute his **score***. By playing a new counter, making an assertion, one alters one's own score, and perhaps that of others.*

Untyped and Types Games

Hypothesis

The game is, at the beginning, untyped, in the sense that specific rules are not given. It is by the action and interaction of several plays that types may emerge: a type is a behavior, that is, in Girard's sense, a set of strategies which behave the same way with regard to other strategies, presented as counter-strategies. Finally, when "typing" the processes, rules emerge, which are rules for building proofs and counter-proofs (that is, globally, paraproofs) in a logical system very close to Linear Logic with Exponentials.

The Assertion Game - I

Let *A* be the speaker who asserts, for instance “The swatch is red”, and let *B* his interlocutor. We must always assume that:

- the **commitment** is undertaken by *A* among a set of possibilities offered by *B*, as entitlements to undertake commitments,
- *A* associates a set (directory) of **entitlements** concerning the way in which *B* can react toward his commitment

The Assertion Game - II

- we may assume *A* and *B* represented each by a *process* which consists of a sequence of actions alternatively positive and negative, so that positive actions of one participant correspond to negative actions of the other.
- as if each of the two locutors, acting like players, had at his own disposal markers (that is ‘sentences’ or propositional contents) to put on *squares* of two sorts: positive and negative.
- moving a marker from a positive square to a negative one is to *commit* oneself in an assertion, while moving a marker from a negative to a positive square is to give an *entitlement* to commit oneself in an assertion.
- Each player chooses his commitment among the entitlements to do so, and each time he or she plays, he or she gives entitlements to the interlocutor. And there may be several rounds in one assertion game.

Arena

Definition

An **arena** $(A, \vdash_A, \lambda_A)$ is given by:

- a **directed acyclic graph** $(A, \vdash_A)$ where:
  - A is the set of **moves**
  - $\vdash_A$ is the **enabling relation** on A. If there is an edge from m to n, we write: $m \vdash_A n$. A move m is said to be **initial** if no other move enables it, we write $\vdash_A m$.
- a function $\lambda_A \ : \ A \rightarrow \{+, -\}$ which labels each move with a polarity.

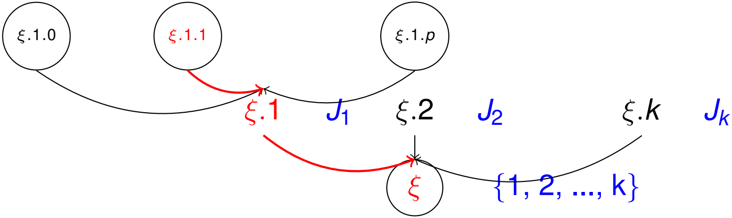
It is assumed that (**alternation** property) if $m \vdash_A n$, then m and n have opposite polarities. If all the initial moves have the same polarity ϵ, we say that ϵ is the polarity of the arena.

The Untyped framework

Definition

A **universal arena** is defined with regard to a **base** (also called an **interface**) Γ which consists of a finite set of disjoint **loci**, provided with polarities in such a way that at most one locus is negative.

- the moves are the special action $\dagger$ and the actions (ξ', I) where every ξ' has some $\xi \in \Gamma$ as prefix, ($\dagger$ and actions (ξ, I) where $\xi \in \Gamma$ are called **initial**)
- the polarity of the initial actions (ξ, I) is the one indicated by the base for ξ, and other polarities are deduced by alternation,
- the enabling relation is such that
  1. $(\xi, I) \vdash (\xi.i, J)$ for all $i \in I$
  2. $x \vdash y$ for each x negative initial action and y positive initial action



ξ positive in the base

for instance, $\Gamma = \sigma \vdash \xi, \eta$

Justified sequences

- The “game of assertion” (*the swatch is red*) is played in an *arena* $A(\xi^+)$, where ξ is the locus of the assertion.
- Among all the possible plays, we distinguish **justified sequences**.

Definition

A **justified sequence** on an arena A is a sequence of actions $\sigma = \sigma_0.\sigma_1....\sigma_n$, with **pointers** between the elements of the sequence which satisfies:

- for each non-initial σ_i, there is a unique pointer to a σ_j ($j < i$) such that $\sigma_j \vdash_A \sigma_i$ (σ_j is called the **justifier** of σ_i),

The example of a dialogue

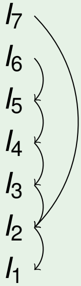
"The Count of Monte-Cristo" by Alexandre Dumas - a dialogue between Abbé Faria (F) and Edmond Dantès (E).

Example

F	Who could benefit from your death? What was your life at this time?	I_1
E	I was ready to become captain of the *Pharaon*; I was about to marry a beautiful young girl.	I_2
F	Was anyone interested in you not becoming the captain of the *Pharaon*?	I_3
E	[. . .], Only one man. [. . .]	I_4
F	Who was he?	I_5
E	Danglars.	I_6
F	Well, tell me about that young girl. . .	I_7

The example of a dialogue

Example



In ludical terms

Example

$$(002, \{0\})$$
$$(001000, \{0\})$$
$$(00100, \{0\})$$
$$(0010, \{0\})$$
$$(001, \{0\})$$
$$(00, \{1, 2\})$$
$$(0, \{0\})$$

Views and strategies

Definition

A **view** (also called **chronicle**) is a justified sequence such that:

- No two following moves have the same polarity
- for each pair of consecutive actions σ_i, σ_{i+1} such that $\lambda(\sigma_i) = +$ and $\lambda(\sigma_{i+1}) = -$, we have $\sigma_i \vdash \sigma_{i+1}$

Definition

A **strategy** is a prefix-closed set of views $\mathcal{D}$ such that:

- if $\sigma.m$ and $\sigma.n \in \mathcal{D}$ and $m \neq n$, then m and n are negative,
- if $\sigma.m$ is maximal in $\mathcal{D}$, then m is positive.

Back to the dialogue example

Example

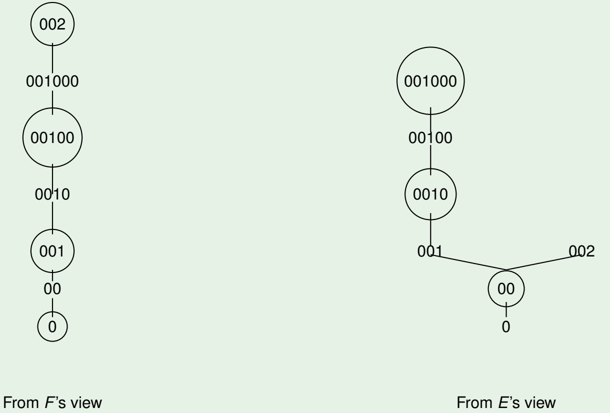


Figure: Edmond and Faria's strategies

Interaction

Example

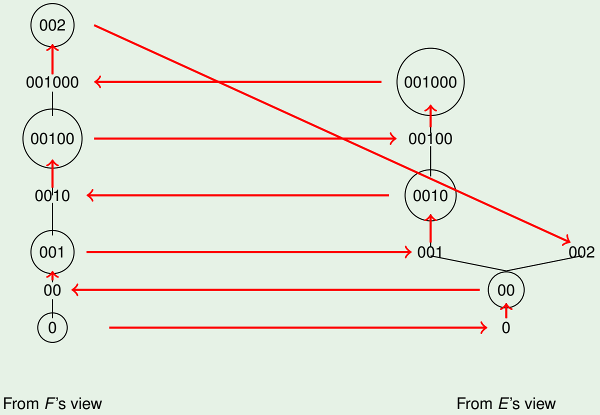
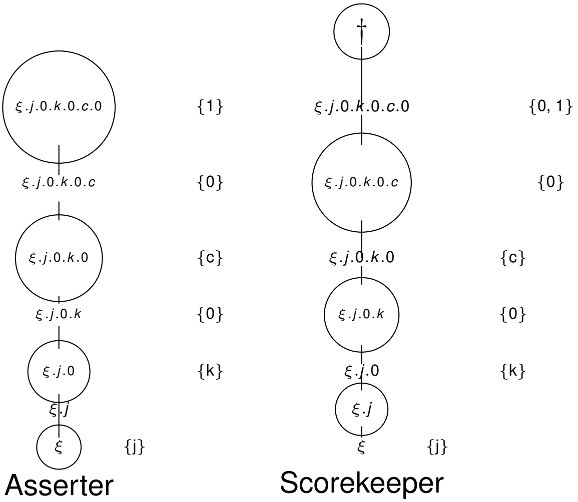


Figure: Edmond and Faria's strategies

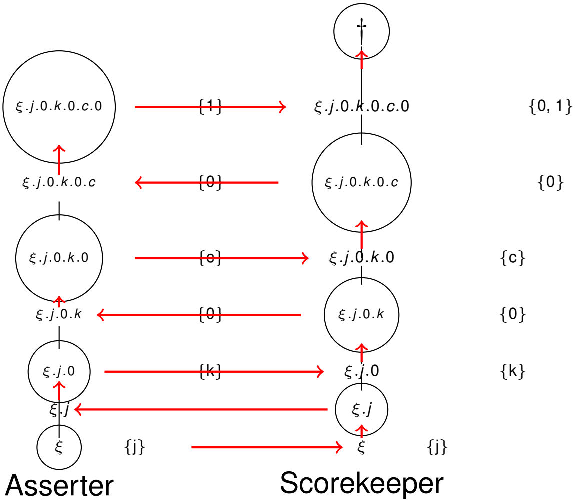
The Assertion game

1. the speaker chooses an object j in some set I_i, which is provided as *entitlements* to address some *theme*, by the interlocutor (even if she is a virtual speaker),
2. having chosen to speak of some definite object (here by means of a singular term), the speaker implicitly entitles his interlocutor to treat this object as concerned by a range of properties, naturally associated with that object,
3. the interlocutor entitles the speaker to choose a property among this range,
4. the speaker chooses a property and entitles his interlocutor to treat this property as concerned by a range of *values*,
5. the interlocutor entitles the speaker to choose a value,
6. the speaker chooses a value and entitles his interlocutor to treat is according to a set of modalities (maybe simply *true* and *false*),
7. the interlocutor entitles the speaker to choose a modality
8. the speaker chooses a modality and waits for an acknowledgement

Assertion game



interaction



Sets of plays

- sets of views $\longrightarrow$ sets of plays
- why a dialogue may fail?

$$\frac{\xi_{i_j} \vdash}{\vdash \xi} \; I_i = \{j\}$$

This corresponds to the entitlement provided by the interlocutor in a negative move:

$$\frac{\vdash \xi_1, ..., \vdash \xi_j, ..., \vdash \xi_n}{\xi \vdash} \; \mathcal{N} = \{\{1\}, ..., \{j\}, ...\{n\}\}$$

if $\{j\} \notin \mathcal{N}$, then the interaction **fails**

The interaction as Dynamics of proofs

$$\dfrac{\vdash \xi_1, \ldots, \quad \dfrac{\dfrac{\ldots, \vdash \xi.j.0.k, \ldots}{\xi.j.0 \vdash}(-, \{\{1\}, \ldots, \{k\}, \ldots, \{m\}\})}{\vdash \xi_j}(+, \xi.j, \{0\}) \quad , \ldots, \vdash \xi_n}{\xi \vdash}\mathcal{N}$$

with, on the locutor's side:

$$\dfrac{\dfrac{\dfrac{\xi.j.0.k \vdash}{\vdash \xi.j.0}(+, \xi.j.0, \{k\})}{\xi.j \vdash}(-, \{\{0\}\})}{\vdash \xi}(+, \xi, \{j\})$$

Then, the interlocutor still records the answer and continues the interaction by providing the range of values and so on

Affirmation, negation and modalities

It is important here to note the last action which consists of selecting a focus (here the colour which is chosen, $c.0$), then selecting a supplementary digit, here limited to 0 or 1. We may assume for instance that 0 is the **negation** of the predicate, and 1 its **affirmation**. We may also envisage a wider range of digits, for instance expressing **modalities**.

Strategies → objects similar to proofs

- DAÏMON

$$\frac{}{\vdash \Delta}\dagger$$

- POSITIVE RULE

$$\frac{(\xi.i \vdash \Delta_i)_{i \in I}}{\vdash \Delta, \xi}(\xi, I)$$

where I is a finite set of integers (maybe empty) and the Δ_i's are pairwise disjoint and included in Δ.

- NEGATIVE RULE

$$\frac{(\vdash (\xi.i_j)_{i_j \in I}, \Delta_I)_{I \in \mathfrak{N}}}{\xi \vdash \Delta}(\xi, \mathfrak{N})$$

$\mathfrak{N}$ is a set (maybe empty or infinite) of finite sets of integers and the Δ_I's, not necessary disjoint, are contained into Δ.

Interpretation

- POSITIVE : the rule of **commitment**. Every time we are entitled to some propositional content assigned to a positive *locus* ξ, we may assign this content to negative *sub-loci* $\xi.i$, for $i \in I$. This has the result to *focus* on this content and to commit oneself in some judgement or question[1].
- NEGATIVE : the rule of **entitlement**. It gives a family of ranges $\mathfrak{N}$ associated with a locus, and it says that every time a focus has been chosen in an assertional move, that *entitles* the other speaker to choose a range of values (or a "property") associated with the object which is assigned to this focus.
- DAIMON : **acknowledgement**

[1] If I is not a singleton, several aspects are dealt with. The interaction can explore all these aspects one after the other.

From a logical point of view

$$\frac{A_{i1} \vdash \Gamma_1 \quad \ldots A_{in_i} \vdash \Gamma_{n_i}}{\vdash (A_{11}^{\perp} \otimes \cdots \otimes A_{1n_1}^{\perp}) \oplus \cdots \oplus (A_{p1}^{\perp} \otimes \cdots \otimes A_{pn_p}^{\perp}), \Gamma}$$

$$\frac{\vdash A_{11}, \ldots, A_{1n_1}, \Gamma \quad \ldots \quad \vdash A_{p1}, \ldots, A_{pn_p}, \Gamma}{(A_{11}^{\perp} \otimes \cdots \otimes A_{1n_1}^{\perp}) \oplus \cdots \oplus (A_{p1}^{\perp} \otimes \cdots \otimes A_{pn_p}^{\perp}) \vdash \Gamma}$$

$$\frac{A \vdash B, \Delta \qquad B \vdash \Gamma}{A \vdash \Delta, \Gamma}$$

where $\cup\Gamma_k \subset \Gamma$ (note that if $\cup_k\Gamma_k$ is strictly included in Γ, this allows to retrieve weakening) and, for $k, l \in \{1, \ldots n_i\}$, $\Gamma_k \cap \Gamma_l = \emptyset$.

The Daimon

instead of axioms:

$$\frac{}{\vdash \Gamma}$$

- a paralogism
- $\longrightarrow$ paraproofs

Interpreting branching nodes

Consequences:

- positive branching node $\longrightarrow \otimes$ (several aspects to be addressed)
- negative branching node $\longrightarrow$ & (a choice of only one aspect among several)

strategies where &-choices have been eliminated = *slices*

The Asserter

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\xi.j.0.k.0.0.0.0 \vdash}{\vdash \xi.j.0.k.0.0.0}\ (\textit{chooses false})\ (+,\{0\})}{\xi.j.0.k.0.0 \vdash}\ \textit{sym accepted}}{\vdash \xi.j.0.k.0}\ (\textit{chooses sym})\ (+,\{0\})}{\xi.j.0.k \vdash}\ \textit{s/a accepted}}{\vdash \xi.j.0}\ (\textit{chooses s/a})}{\xi.j \vdash}\ \textit{j accepted}}{\vdash \xi}\ (\textit{chooses j})$$

the speaker chooses the value *sympathetic* with negative modality 0, and then stops.

Results in : *she is not sympathetic*

The scorekeeper

$$
\dfrac{\ldots \qquad \dfrac{\dfrac{\ldots \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash \xi.j.0.k.0.0.0.0}\dagger \qquad \dfrac{}{\vdash \xi.j.0.k.0.0.0.1}\dagger}{\xi.j.0.k.0.0.0 \vdash}(-,\{\{0\},\{1\}\})}{\vdash \xi.j.0.k.0.0} \qquad \dfrac{\ldots}{\vdash \xi.j.0.k.0.1}}{\xi.j.0.k.0 \vdash}(-,\{\{0\},\{1\}\})}{\vdash \xi.j.0.k} \qquad \ldots}{\xi.j.0 \vdash}}{\vdash \xi.j} \qquad \ldots}{\xi \vdash}
$$

On the contrary...

An illegal move...

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\color{red}{\xi.j.0.k.0.1} \vdash}{\vdash \xi.j.0.k.0.0.0.0.0}\ (+, \xi.j.0.k.0, \{1\})}{\xi.j.0.k.0.0.0.0 \vdash}}{\vdash \xi.j.0.k.0.0.0}}{\color{red}{\xi.j.0.k.0.0} \vdash}}{\vdash \color{blue}{\xi.j.0.k.0}}\ (+, \xi.j.0.k.0, \{0\})}{\xi.j.0.k \vdash}}{\vdash \xi.j.0}}{\xi.j \vdash}}{\vdash \xi}$$

Need to replay (backtracking)

- If the scorekeeper's viewpoint stays the same, **normalization** would lead to change the "slice" of the paraproof, playing on a branch which, after the previous normalization steps, has been removed. In ordinary Ludics, normalization fails in such a case.

In Ludics **with repetitions** (which amounts to **adding exponentiels**), that would be possible.
cf. Balsadella & Faggian (2009), K. Ranalter (september 2010), ...

Conclusion

- what are we *doing* when we use some specific words or expressions? (*but, on the contrary, ...*), what *move* in a play? (cf. O. Ducrot, 1984)
- **typing**: the result of interactions (type = behaviour = $\mathcal{D}^{\perp\perp}$, where $\mathcal{E}^{\perp} = \{\mathcal{G}; \mathcal{E} \perp \mathcal{G}\}$)
- **material implication = subtyping**, $A \Rightarrow_m B$ iff $\mathcal{A}^{\perp\perp} \subset \mathcal{B}^{\perp\perp}$ or $\mathcal{B}^{\perp} \subset \mathcal{A}^{\perp}$ (every counter-strategy for $\mathcal{B}$ is also a counter-strategy for $\mathcal{A}$)

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